Erratum: Guess Free Maximization of Submodular and Linear Sums

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1 Introduction

An error was found by Xin Sun and Xianzhao Zhang in the proof of Theorem 3 in my paper "Guess Free Maximization of Submodular and Linear Sums" [1]. This document desribes this error, as well as some additional minor errors in the same paper, and then explains how all these errors can be fixed. I would like to thank Xin Sun and Xianzhao Zhang for pointing to me the error in the proof of Theorem 3.

2 Errors in the Statements of Theorems 2 and 3

Theorems 2 and 3 describe hardness results involving a linear function ℓ. Both theorems state the range of ℓ as $\mathbb{R}_{\geq 0}$, which incorrectly implies that ℓ is always non-negative. However, the true sign of ℓ depends on the parameter r of these theorems. More specifically, ℓ is non-negative when r is non-negative, and non-positive when r is non-positive (when r is zero, ℓ is the zero function).

In addition to the above, Theorem 3 is stated twice in the paper. Once on Page 859, and once immediately before its proof on Page 870. Unfortunately, the restatement on Page 870 is incorrect (it is in fact a restatement of Theorem 2 rather than Theorem 3).

3 Error in the Proof of Theorem 3

The final step in the proof of Theorem 3 is showing that for a particular random set S and set function h, it holds that $\mathbb{E}[h(S)] \geq \mathbb{E}[1 - e^{|S|/k} + \varepsilon]$, where k and ε are given positive numbers. Towards this goal, the proof shows that

$$\mathbb{E}[h(S)] \geq \max_{\lambda \in [0,1]} \{(1 - e^{-\lambda} + \varepsilon) + \lambda r\} - (r/k) \cdot \mathbb{E}[|S|] \ , \tag{1}$$

where r is another given number. The proof then argues the the right hand side of the last inequality is at least $\mathbb{E}[1 - e^{|S|/k} + \varepsilon]$. This argument is done by considering a few cases based on the value of r. The case in which an error was found is the case of $r \in [-1, -1/e]$. In this case one can choose $\lambda = -\ln(-r)$, to get that the right hand side of Inequality (1) is at least

$$1 + r + \varepsilon - r \cdot \ln(-r) - (r/k) \cdot \mathbb{E}[|S|] = 1 + \mathbb{E}\left[r - r \cdot \ln(-r) - \frac{r \cdot |S|}{k}\right] + \varepsilon \ . \tag{2}$$

To lower bound the last expression, the proof then considers the derivative of the expression within the expectation. Unfortunately, the derivative was not calculated correctly in the original proof.

The correct derivative is

$$1 - \ln(-r) - r \cdot \frac{1}{r} - \frac{|S|}{k} = -\ln(-r) - \frac{|S|}{k} \ ,$$

which is an increasing function of r that takes the value of 0 at $r = -e^{-|S|/k}$. Therefore, Equation (2) is minimized for this value of r, and since Equation (2) lower bounds the right side of Inequality (1), this implies

$$\begin{aligned}\mathbb{E}[h(S)] &\geq 1 + \mathbb{E}\left[-e^{-|S|/k} + e^{-|S|/k} \cdot \ln(e^{-|S|/k}) + \frac{e^{-|S|/k} \cdot |S|}{k}\right] + \varepsilon \\ &= 1 + \mathbb{E}[-e^{-|S|/k}] + \varepsilon = \mathbb{E}[1 - e^{-|S|/k} + \varepsilon] \ ,\end{aligned}$$

which is what we needed to prove.

References

[1] Moran Feldman. Guess free maximization of submodular and linear sums. *Algorithmica*, 83(3):853–878, 2021.